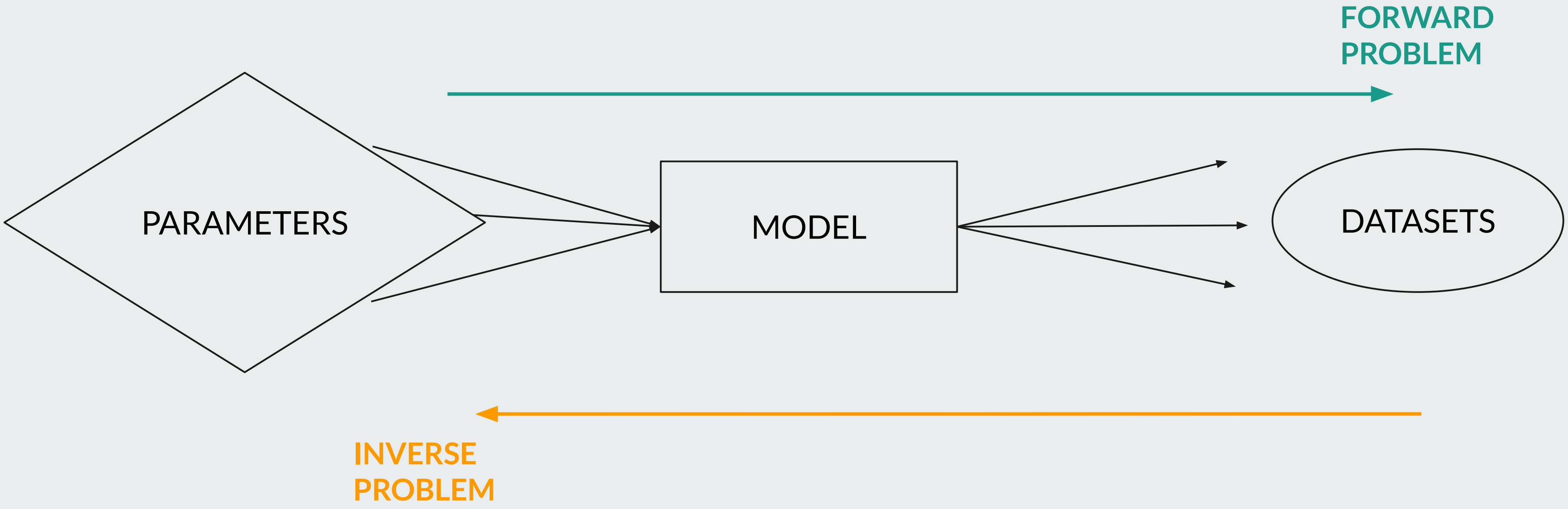
OK so far, you have a model and you can generate predictions given some parameters:



We can adopt a **qualitative** approach:

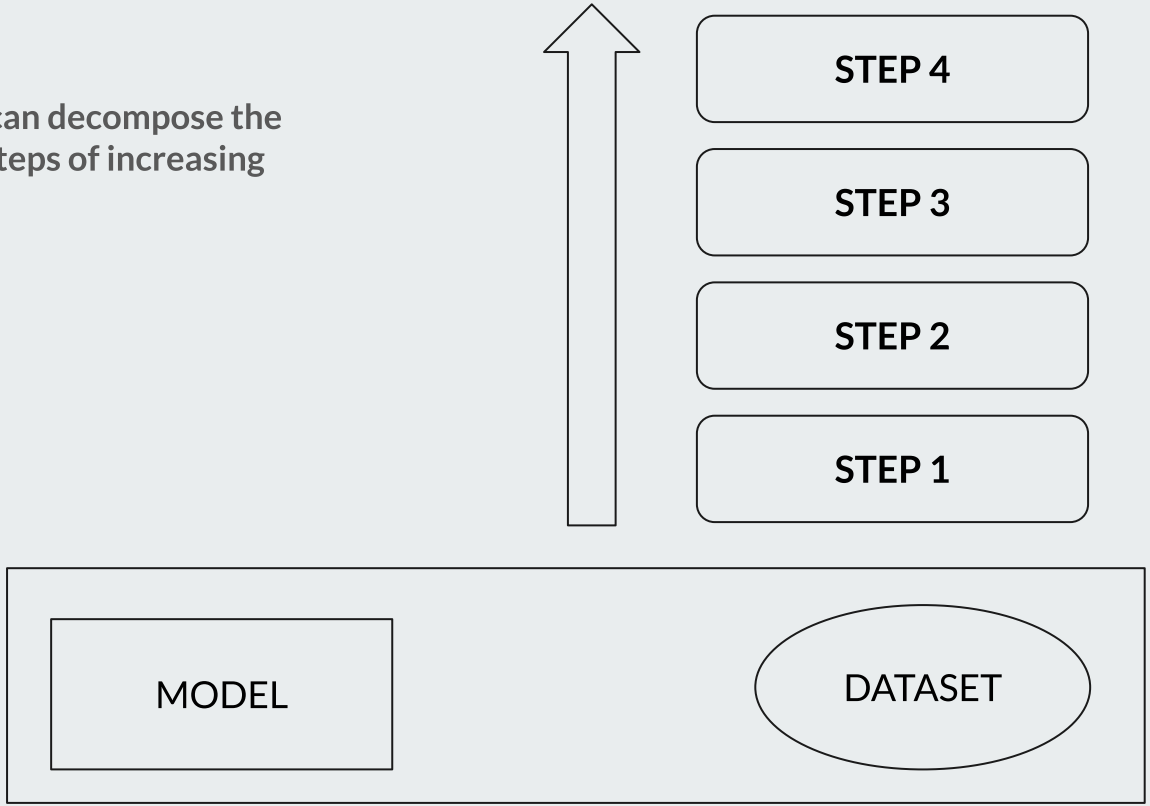
– Can my model reproduce some particular aspects of my dataset? (e.g. a positive relationship between two quantities, a hump-shaped trend for some variable…)

– For what range of parameters does this happen?

Generalized models can be useful here (e.g. Yeakel et al. 2011 *Theor. Ecol.*)

Or we we can adopt a more **quantitative** approach (stats)

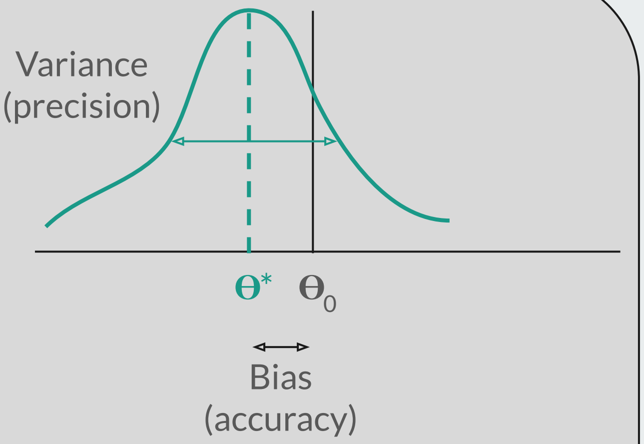
For simplicity, we can decompose the process into four steps of increasing ambition:



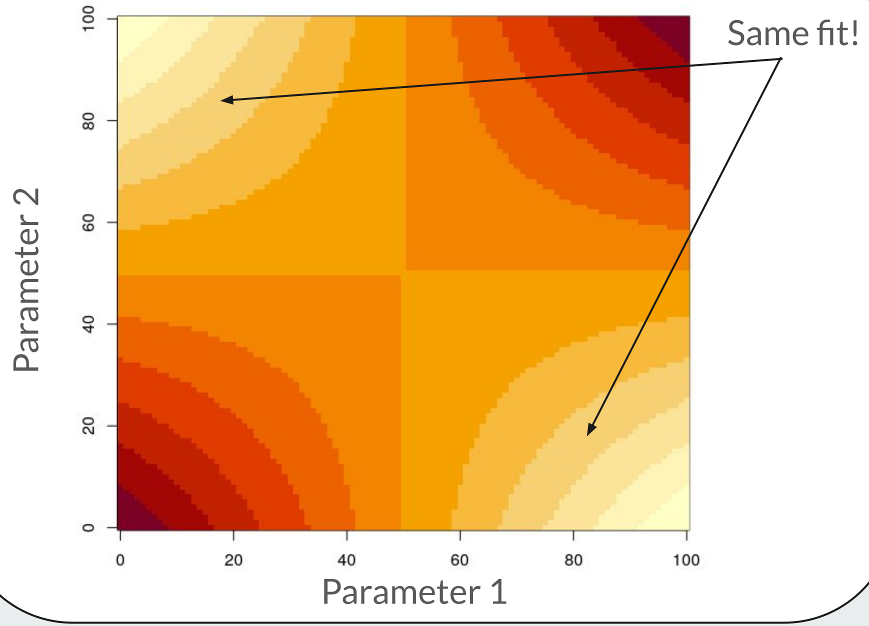
STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?

Major problems:
– we may obtain **biased** estimates
– the model may not be **identifiable**

STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?



Total error = Bias^2 + Variance



STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?

Common approaches:

– minimize some **distance** between predictions and data (e.g. least squares)

– maximize **likelihood**

– maximize **posterior probability** (Bayesian approaches)

STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?

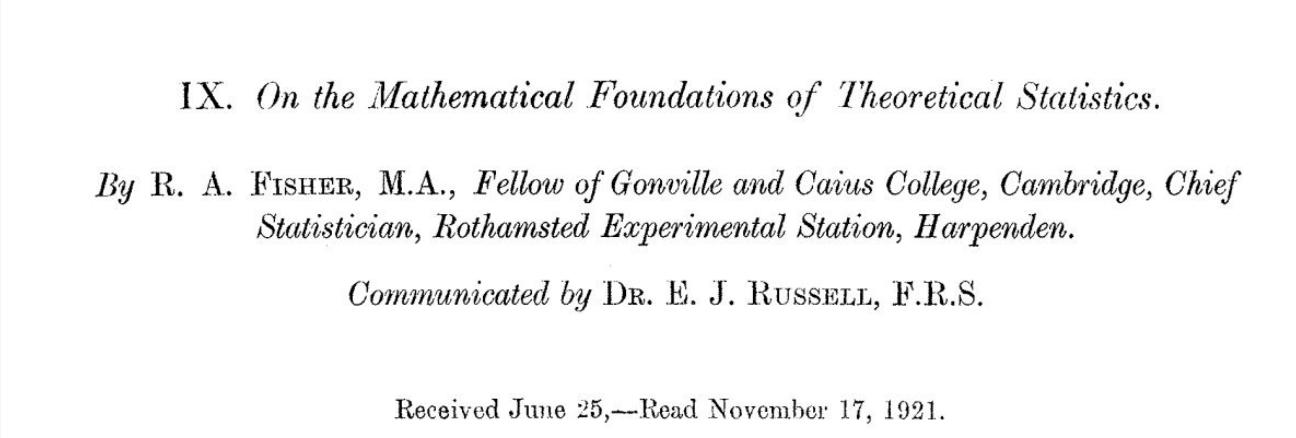
Least squares:

+ flexible, robust enough, very fast minimization techniques
+ equivalent to ML under certain assumptions*
- not applicable to all models
- somewhat ad-hoc: other distances could be used (absolute differences…)

Maximum likelihood:

+ fully general, intuitive, solid theoretical grounding
+ consistent (asymptotically unbiased… *if model is true*)
- can be hard to compute and maximize

STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?



IX. *On the Mathematical Foundations of Theoretical Statistics.*

By R. A. FISHER, M.A., *Fellow of Gonville and Caius College, Cambridge, Chief Statistician, Rothamsted Experimental Station, Harpenden.*

Communicated by DR. E. J. RUSSELL, F.R.S.

Received June 25,—Read November 17, 1921.

STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?

Least squares:

- \+ flexible, robust enough, very fast minimization techniques
- \+ equivalent to ML under certain assumptions*
- \- not applicable to all models
- \- somewhat ad-hoc: other distances could be used (absolute differences…)

Maximum likelihood:

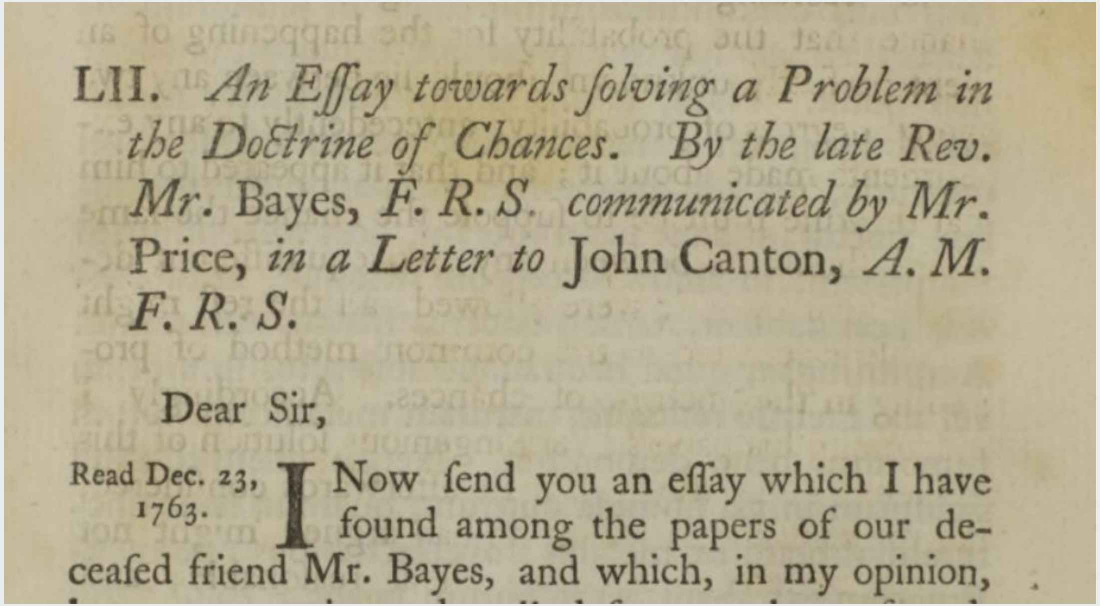
- \+ fully general, intuitive, solid theoretical grounding
- \+ consistent (asymptotically unbiased… *if model is true*)
- \- can be hard to compute and maximize

Posterior probability (Bayesian approaches):

- \+ supplements ML with **prior knowledge** on parameter values
- \+ efficient sampling algorithms (priors guide the exploration of parameter space)
- \+ prior distributions can alleviate non-identifiability issues
- \- supplements ML with **prior knowledge** on parameter values
- \- can be slow to converge

STEP 1: how to find the best parameter values to fit the data (point estimation, model fitting)?

Bayes (1763)
Phil. Trans. Royal Soc.



LII. *An Eſſay towards ſolving a Problem in the Doctrine of Chances. By the late Rev.* Mr. Bayes, *F. R. S. communicated by Mr.* Price, *in a Letter to* John Canton, *A. M. F. R. S.*

Dear Sir,

Read Dec. 23, 1763.

I Now ſend you an eſſay which I have found among the papers of our deceaſed friend Mr. Bayes, and which, in my opinion,

STEP 2: how to quantify the uncertainty in parameter estimates, and the quality of the fit (goodness of fit)?

Major problems:
– we want to construct good **intervals** around parameter estimates: there are many of them
– with not so much data, we may have very little **statistical power** to evaluate the goodness of the fit (and failing to reject is not accepting)

STEP 2: how to quantify the uncertainty in parameter estimates, and the quality of the fit (goodness of fit)?

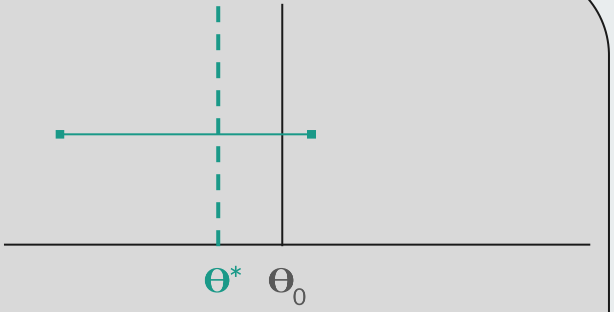
Common approaches (uncertainty):

– resample dataset/refit model (bootstrap, jacknife…)

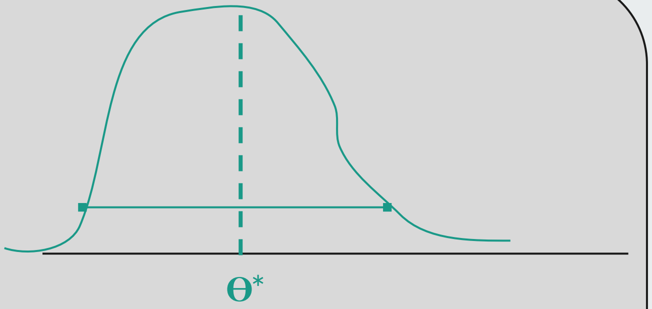
– use likelihood surface theory to get confidence intervals (Fisher information…)

– use Bayesian approaches to compute credible intervals

STEP 2: how to quantify the uncertainty in parameter estimates, and the quality of the fit (goodness of fit)?



x% **confidence interval**: if you use this procedure many times, the true value θ_0 will fall in your intervals x% of the time



x% **credible interval**: I have x% of my posterior credence (given the prior) contained in this interval

STEP 3: how to compare different models together and select the 'best' (model selection)?

Major problems:
– a more complex model will always fit the data better but…
– the **bias/variance trade-off**, or the **curse of complexity**: for a given amount of data, too simple a model will have little variance/high bias (**underfitting**), too complex a model will have low bias/huge variance (**overfitting**)
– in both cases we have poor estimation of parameters (total uncertainty = bias^2 + variance)
– in both cases, we'll have poor prediction power for future/other datasets
– we must find some intermediate level of complexity, i.e. make some **compromise**

STEP 3: how to compare different models together and select the ‘best’ (model selection)?

Common approaches:

– Split dataset (**cross validation**, training/test datasets…)

– is a particular parameter ‘significant’ or not? (model simplification)

– **Likelihood ratio tests**

– Information criteria for model comparison (**AIC…**)

– Regularization or penalization techniques (ridge regression, LASSO…)

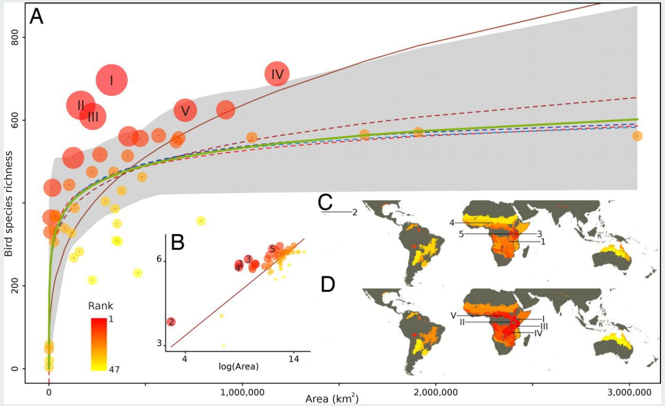
Tredennick et al. (2021) *Ecology*

STEP 4: how to use several alternative models rather than just one (multimodel inference)?

Major problems:
– How to reduce model selection bias (see Freedman's paradox)?
– How to include model selection uncertainty?
– How to combine the estimates or predictions from different competing models, and combine them in an optimal way?

STEP 4: how to use several alternative models rather than just one (multimodel inference)?

Common approaches:
– Take all models and do some ad-hoc **consensus** (e.g. average or median prediction)
– Use **model-averaging** techniques



Dormann et al. (2018) *Ecological Monographs*

SAR: Guilhaumon et al. (2008) *PNAS*
TPC: Padfield et al. (2021) *Methods in Ecology & Evolution*