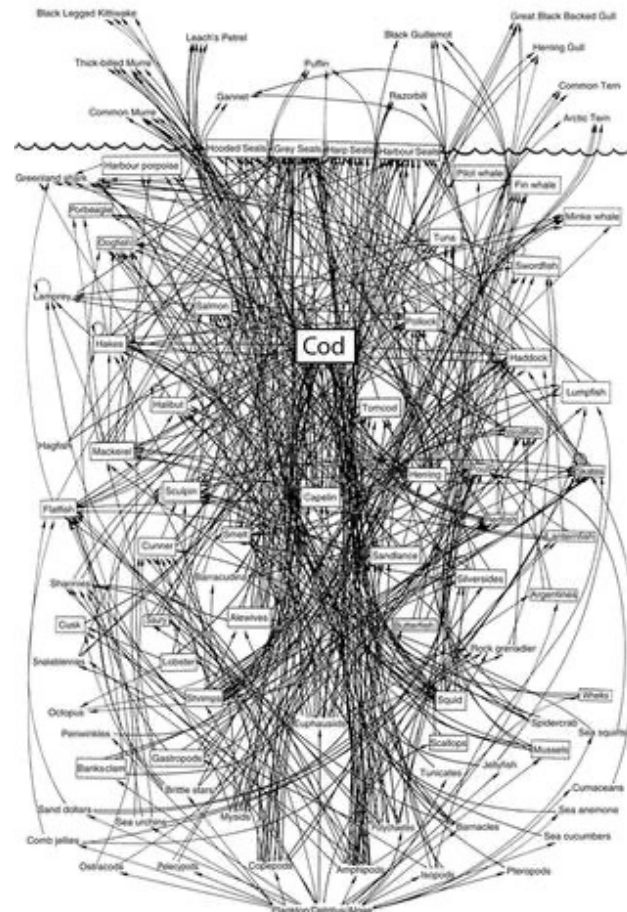


Complexity and stability of empirical food webs

Food webs

Complex systems

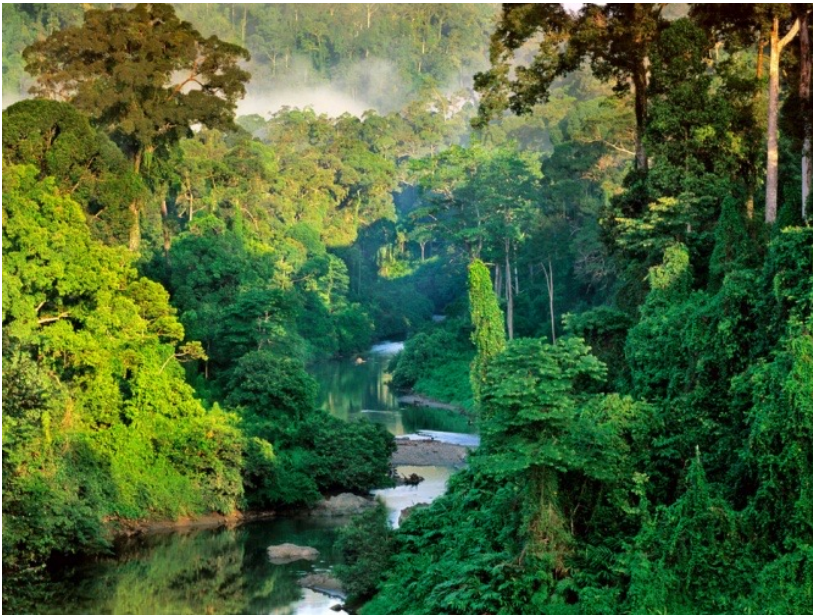


Will the ecosystem buffer or amplify a perturbation ?

The complexity stability debate

Until the 70's:

Diversity stabilizes ecosystems (Odum 1953, MacArthur 1955)



Guyane, tropical forest.



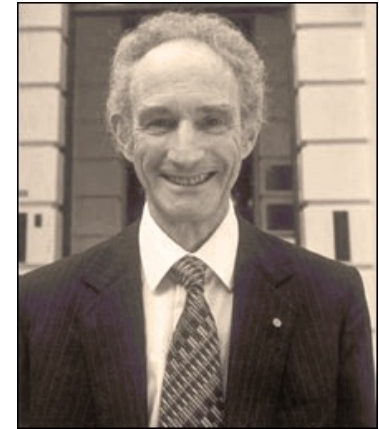
Alaska, boreal forest.

The complexity stability debate

Complexity decreases ecosystem stability (May 1972)

Complexity:

- Species richness S
- Connectance C
- Variance of interaction strengths σ^2



Professor Lord Robert May

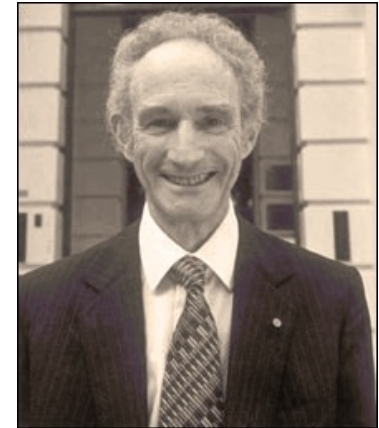
Stability criterion: $\sigma\sqrt{SC} < 1$

The complexity stability debate

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Professor Lord Robert May

Stability criterion: $\sigma\sqrt{SC} < 1$

Where does this result come from?

Local stability analysis

Asymptotic stability: rate to which species populations go back to their initial densities after a pulse disturbance

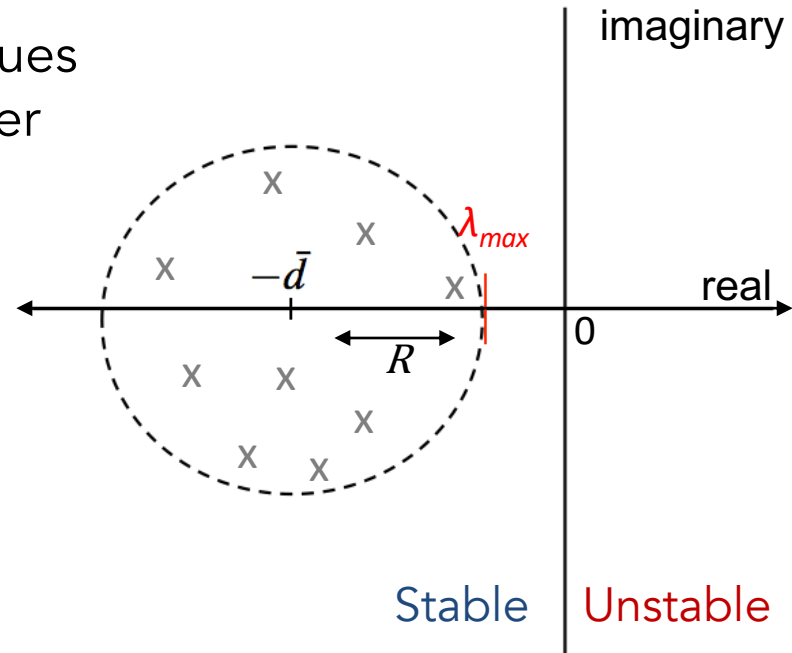


Largest eigenvalue of the community matrix (its real part)

The circular law

On a complex plan: all the eigenvalues are contained in a circle of center $(-d, 0)$ and radius R .

The system is stable if $\text{Re}(\lambda_{\max}) < 0$.

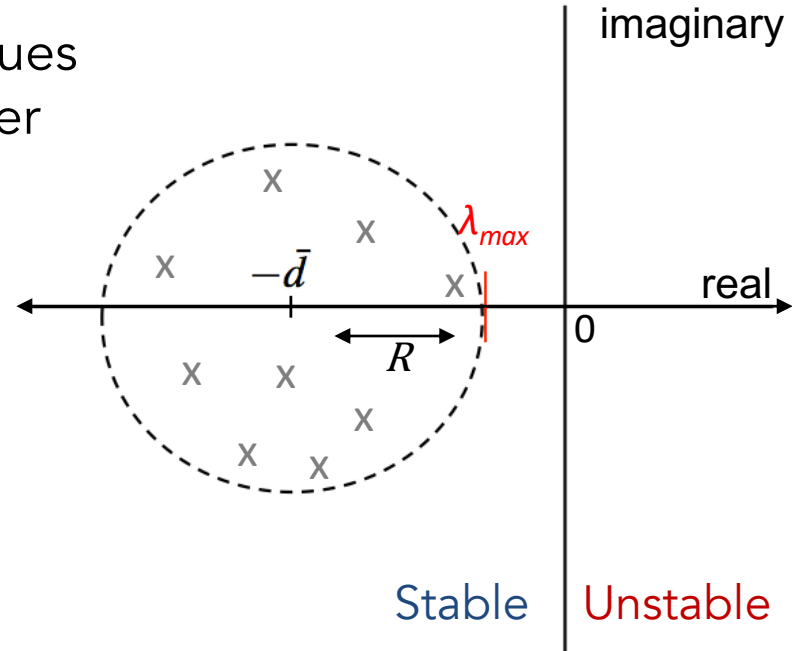


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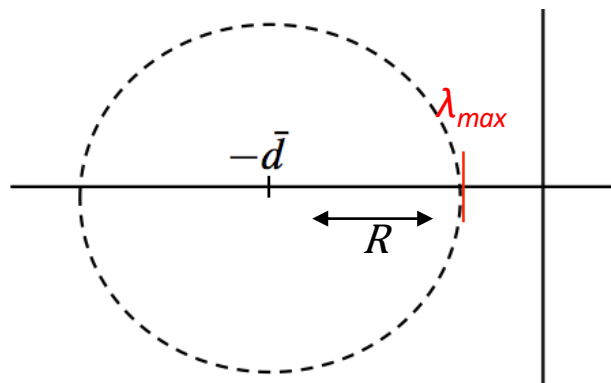
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Stability criterion: $R < d$



The circular law

Link the stability of a matrix to its properties



$$C = \begin{pmatrix} -d_1 & +c_{2,1} & +c_{3,1} & 0 \\ -c_{1,2} & -d_2 & 0 & +c_{4,2} \\ -c_{1,3} & 0 & -d_3 & +c_{4,3} \\ 0 & -c_{2,4} & -c_{3,4} & -d_4 \end{pmatrix}$$

\bar{d} : mean of diagonal terms of C

In random communities: $R = \sigma\sqrt{SC}$

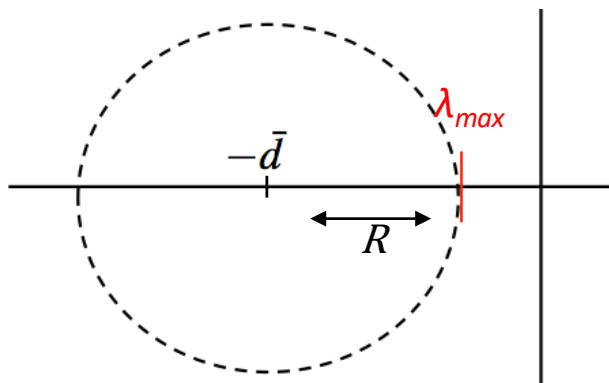
σ^2 : variance of non-diagonal terms c_{ij}

S : size of the matrix

C : proportion of non-zero terms

The circular law

Link the stability of a matrix to its properties



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\bar{d} : mean of diagonal terms of C (magnitude of density dependence)

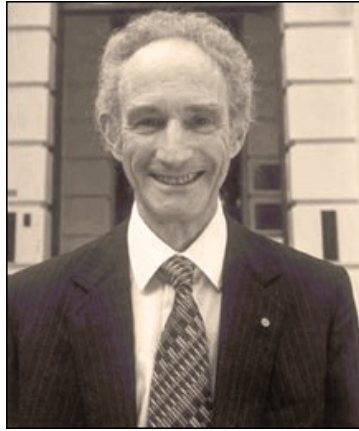
In ecological communities : $R = \sigma\sqrt{SC}$

σ^2 : variance of interaction strengths

S : species richness

C : connectance

Let's check this result in R



« In short, there is no comfortable theorem assuring that increasing diversity and complexity beget community stability; rather, as a mathematical generality the opposite is true.

The task, therefore, is to elucidate the devious strategies which make for stability in enduring natural systems. » (May 2001).

1. What is the actual complexity-stability relationship in empirical communities?

1. What is the actual complexity-stability relationship in empirical communities?
2. What are the « *devious strategies* » of real communities that allow them to persist despite their complexity?

Food-web dataset

116 quantitative food webs from Ecopath models
(Christensen 1992)

For each species i :

- biomass B_i (tons/km²)
- *production* $(P/B)_i$ (year⁻¹)
- *consumption* $(Q/B)_i$ (year⁻¹)
- diet composition DC_{ji}

Assumption: food webs are at equilibrium

Derivation of interaction strengths from data

What we want:

α_{rc} : negative effect of consumer c on resource r

α_{cr} : positive effect of resource r on consumer c

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What is measured:

- biomass B_i (tons/km²)
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- diet composition DC_{ji}

From De Ruiter et al. (1995):

$$\alpha_{rc} = - (DC_{cr} \times (Q/B)_c) / B_r$$

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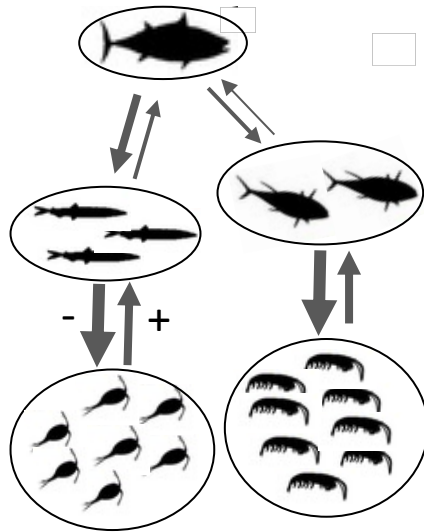
$$\alpha_{cr} = \alpha_{rc} \times e_{rc}, \text{ with } e_{rc} = (P/B)_c / (Q/B)_c$$

$$\rightarrow \alpha_{cr} \leq \alpha_{rc}$$

Practice in R

1. computing food-web complexity and stability
2. analysing the relationship between complexity and stability in empirical food webs
3. comparing the complexity-stability relationship of empirical and « randomized » food webs

What are the non-random properties of food webs?



$$M = \begin{pmatrix} -d_1 & +c_{2,1} & +c_{3,1} & 0 \\ -c_{1,2} & -d_2 & 0 & +c_{4,2} \\ -c_{1,3} & 0 & -d_3 & +c_{4,3} \\ 0 & -c_{2,4} & -c_{3,4} & -d_4 \end{pmatrix}$$

- H_1 : food web topology (who eat whom)
- H_2 : correlation between $+c_{ji}$ and $-c_{ij}$
- H_3 : interaction strength distribution

